Precise Magnetic Field Modeling Techniques of Rotation Problems Using Transient Finite-Element Method

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Abstract —A general and precise method to model rotary machines using transient magnetic field finite-element method (FEM) is presented. The merits of the proposed methods are that the nonlinear iteration on the rotor position is unnecessary and its accuracy is guaranteed. A curvilinear element is used in the two sides of the sliding surface to reduce the numerical noise arising from mesh rotation. The proposed methods to deal with matching boundary conditions and periodic boundary conditions are very general, accurate and flexible.

I. INTRODUCTION

For simulation of rotary machines, it is necessary to deal with issues concerning the relative movements between stator and rotor. If the mechanical balance equation is coupled into the system equation, the rotor position is an unknown during nonlinear iterations at each step [1]. This means the stator mesh and rotor mesh should be re-connected at each step of nonlinear iteration. Such requirement is very complex in implementation and may also lead to divergence. The traditional method is to use the last step's rotor position as the rotor position of the current step. However there is a one step delay in the rotor position with this approach. If triangular or quadrilateral element is used and the rotor moves, the mesh elements in the two sides of the sliding interface may also overlap, albeit only slightly, and numerical errors will appear. In order to deal with the connection of the stator and rotor meshes, several methods such as shifting the nodes, modifying the mesh or re-meshing the airgap can be employed. A macro element in the airgap can also be used [2]. The demerit is that a special formulation for the airgap should be developed. Alternatively, one can map the nodes on one side (say in the rotor) to the other side (say in the stator) [3]. The challenge of this method is to find the best means in order to retain an accurate continuity of the fields between the two sides of the sliding surface.

In this paper, a general method to model the rotational problems is presented. The rotor's position is predicted from the last step and the predicted position has a third-order precision; after the solution of the current step is obtained, the position is then modified. By using this method, the rotor's position is pre-determined before each time step and its accuracy is guaranteed. A curvilinear element is used along the sliding surface of the rotary movement. The number of nodes on the periodic boundary conditions can be different. Between the moving objects and the stationary objects, a matching boundary method is used. A general, accurate and flexible method to determine the transformation matrix between the slave and master nodes is presented. The merit of this method is that the position of the points on the sliding surface, on which the solutions on the two sides are kept the same, can be chosen arbitrarily and are independent of the nodes of the meshes.

II. PREDICTION OF ROTOR POSITION

The rotor position θ^k is predicted from the last step. After the field is solved, the corrected values of acceleration, speed and position are:

$$\begin{cases} a^k = \left\{ \left(T_{em} - T_{load} \right) - \left[\omega^{k-1} + \frac{1}{2}(\Delta t^k) a^{k-1} \right] \lambda \right\} \Big/ \left[J + \frac{1}{2}\lambda(\Delta t^k) \right] \cdot \\ \omega^k = \omega^{k-1} + (\Delta t^k) \dfrac{(a^k + a^{k-1})}{2} \\ \theta^k = \theta^{k-1} + \Delta t^k \omega^{k-1} + \dfrac{(\Delta t^k)^2}{2} a^{k-1} + \dfrac{(\Delta t^k)^2}{4} (a^k - a^{k-1}) \end{cases} \quad (1)$$

The corrected value of the position at the current time step is used to predict the new position for the next $(k+1)^{th}$ time step:

$$\theta^{k+1} \approx \theta^k + \Delta t^{k+1} \omega^k + \frac{(\Delta t^{k+1})^2}{2} a^k + \frac{(\Delta t^{k+1})^3}{4} \frac{(a^k - a^{k-1})}{(\Delta t^k)} \ . \quad (2)$$

The proposed method has a higher order precision when compared to that from traditional methods using the formula.

III. CURVILINEAR ELEMENT FOR SLIDING SURFACE

Curvilinear elements on the two sides of the sliding surface are proposed. Using isoparametric second-order curvilinear elements, six points are required to transform the global coordinates to local coordinates. This allows the edges of the elements to be curvilinear. The coordinates and the magnetic potential are modeled by the same shape function.

IV. A GENERAL METHOD FOR TRANSFORMATION MATRIX OF MATCHING BOUNDARIES

For simplicity, a simple case as shown in Fig. 1 is taken as an example to describe the proposed method. Between the moving objects and the stationary objects, there is a sliding surface. If the nodes are on the moving objects on the sliding surface, these nodes are referred as master nodes; if the nodes belong to the stationary objects, these nodes are referred as the slave nodes. The variables on the master nodes will be solved; the variables on the slave nodes are dependent on those on the mater nodes.

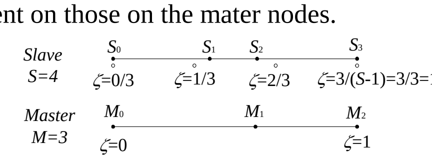

Fig. 1. A matching boundary condition.

Supposing there are 4 nodes on the slave surface, the magnetic potential on any position on the slave surface is

$$A_S(\zeta) = N_{S0}(\zeta)A_{S0} + N_{S1}(\zeta)A_{S1} + N_{S2}(\zeta)A_{S2} + N_{S3}(\zeta)A_{S3} \,. \quad (3)$$

Four points are uniformly taken on the slave surface. On these 4 points, one has

$$\begin{Bmatrix} A_S(0) \\ A_S\left(\frac{1}{3}\right) \\ A_S\left(\frac{2}{3}\right) \\ A_S(1) \end{Bmatrix} = \begin{bmatrix} N_{S0}(0) & N_{S1}(0) & N_{S2}(0) & N_{S3}(0) \\ N_{S0}\left(\frac{1}{3}\right) & N_{S1}\left(\frac{1}{3}\right) & N_{S2}\left(\frac{1}{3}\right) & N_{S3}\left(\frac{1}{3}\right) \\ N_{S0}\left(\frac{2}{3}\right) & N_{S1}\left(\frac{2}{3}\right) & N_{S2}\left(\frac{2}{3}\right) & N_{S3}\left(\frac{2}{3}\right) \\ N_{S0}(1) & N_{S1}(1) & N_{S2}(1) & N_{S3}(1) \end{bmatrix} \begin{Bmatrix} A_{S0} \\ A_{S1} \\ A_{S2} \\ A_{S3} \end{Bmatrix} . \quad (4)$$

Similarly, supposing there are 3 nodes on the master surface, the magnetic potential on any position on the master is

$$A_M(\zeta) = N_{M0}(\zeta)A_{M0} + N_{M1}(\zeta)A_{M1} + N_{M2}(\zeta)A_{M2} \,. \quad (5)$$

The four points are still uniformly taken on the master. On these 4 points, one has

$$\begin{Bmatrix} A_M(0) \\ A_M\left(\frac{1}{3}\right) \\ A_M\left(\frac{2}{3}\right) \\ A_M(1) \end{Bmatrix} = \begin{bmatrix} N_{M0}(0) & N_{M1}(0) & N_{M2}(0) \\ N_{M0}\left(\frac{1}{3}\right) & N_{M1}\left(\frac{1}{3}\right) & N_{M2}\left(\frac{1}{3}\right) \\ N_{M0}\left(\frac{2}{3}\right) & N_{M1}\left(\frac{2}{3}\right) & N_{M2}\left(\frac{2}{3}\right) \\ N_{M0}(1) & N_{M1}(1) & N_{M2}(1) \end{bmatrix} \begin{Bmatrix} A_{M0} \\ A_{M1} \\ A_{M2} \end{Bmatrix} . \quad (6)$$

On the sliding surface, one has:

$$\left\{ A_S(0) \quad A_S\left(\frac{1}{3}\right) \quad A_S\left(\frac{2}{3}\right) \quad A_S(1) \right\}^T = \left\{ A_M(0) \quad A_M\left(\frac{1}{3}\right) \quad A_M\left(\frac{2}{3}\right) \quad A_M(1) \right\}^T . \quad (7)$$

That means:

$$[N_S]_{S\times S} \begin{bmatrix} A_{S0} \\ A_{S1} \\ A_{S2} \\ A_{S3} \end{bmatrix} = [N_M]_{S\times M} \begin{bmatrix} A_{M0} \\ A_{M1} \\ A_{M2} \end{bmatrix}, \quad \begin{bmatrix} A_{S0} \\ A_{S1} \\ A_{S2} \\ A_{S3} \end{bmatrix} = [N_S]^{-1}_{S\times S} [N_M]_{S\times M} \begin{bmatrix} A_{M0} \\ A_{M1} \\ A_{M2} \end{bmatrix} . \quad (8)$$

Each column of the transformation matrix $[N_S]^{-1}_{S\times S}[N_M]_{S\times M}$ can be calculated by solving:

$$[N_S]_{S\times S}[x_0 \quad x_1 \quad x_2 \quad x_3]^T = [N_M]_{S\times M}\big|_{\text{one column only}} \,. \quad (9)$$

A matrix solver which can deal with multi RHS can be used to solve such problem.

V. NUMERICAL EXPERIMENTS

A. A Simple Example to Show Accuracy Improvement

An example in Fig. 2 is used to study the numerical errors from the rotor rotation. To enlarge the numerical error for comparison, a coarse mesh with 4152 triangles is used. When the rotor rotates, the coil should theoretically have no induced electromagnetic force. But because of numerical error, the computed electromagnetic force in the coil is not exactly equal to zero. In this study the rotor rotates at the speed of 360 degrees/s. A small time step size of 0.001s is used to pick up all numerical noises. The computed electromagnetic forces using the proposed method and the traditional method are shown in Figs. 3 and 4, respectively. In both cases the mesh density is the same. It can be seen that the numerical error of the new method is about 130 times less than that of the traditional method.

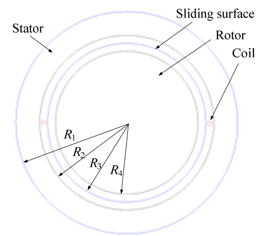

Fig. 2. A simple example to detect the numerical error from mesh rotation.

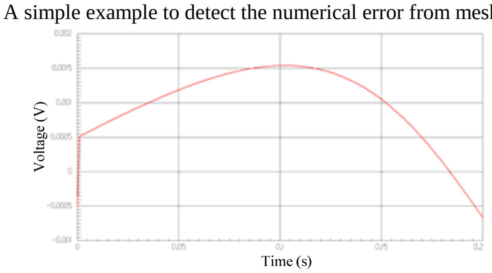

Fig. 3. The computed emf using proposed method.

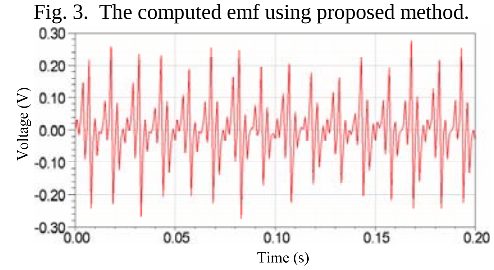

Fig. 4. The computed emf using traditional method.

B. An Example of Induction Motor

The Team Workshops Problem No. 30 [4], which is a three-phase solid-rotor induction motor (IM), is taken as the illustrating example to verify the validity of the proposed method. The calculated phase voltage with the proposed method and the analytical results are compared in Fig. 5. It is shown that the differences between the analytical results and the proposed FEM method are acceptably small.

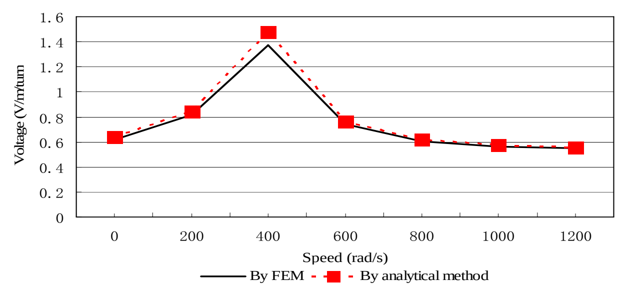

Fig. 5. Comparison of emf results.

VI. REFERENCES

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